Designing (Optimal) Multi-dimensional Blockchain Fees

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Fee markets with fixed relative prices are inefficient

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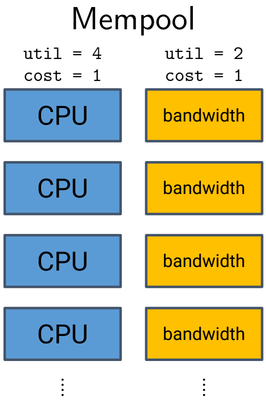
This talk: a framework to optimally set multi-dimensional fees for congestion control

Outline

Fixed relative prices limit throughput



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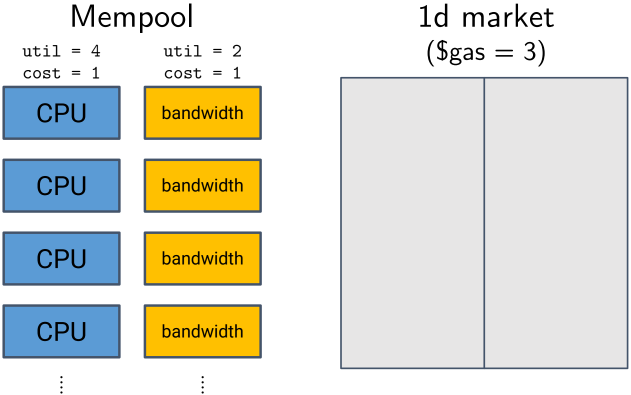

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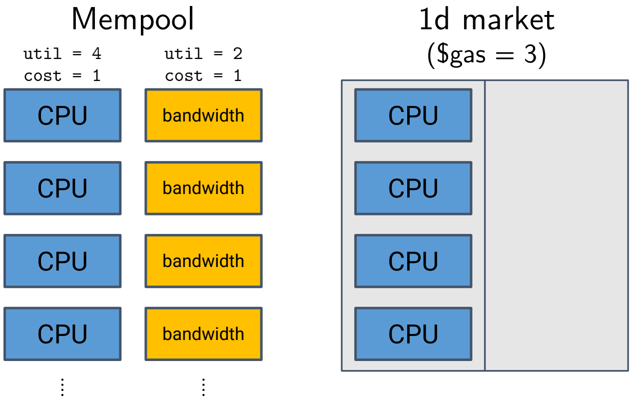

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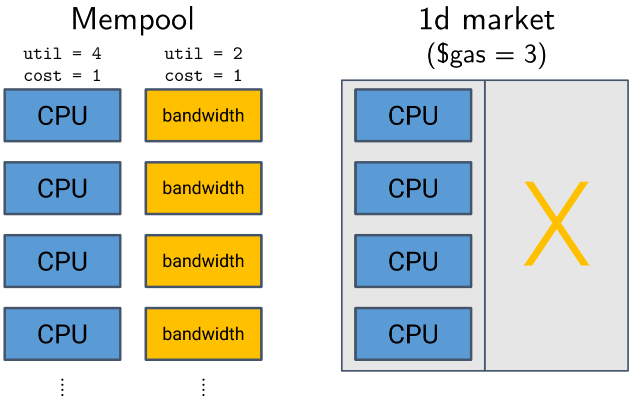

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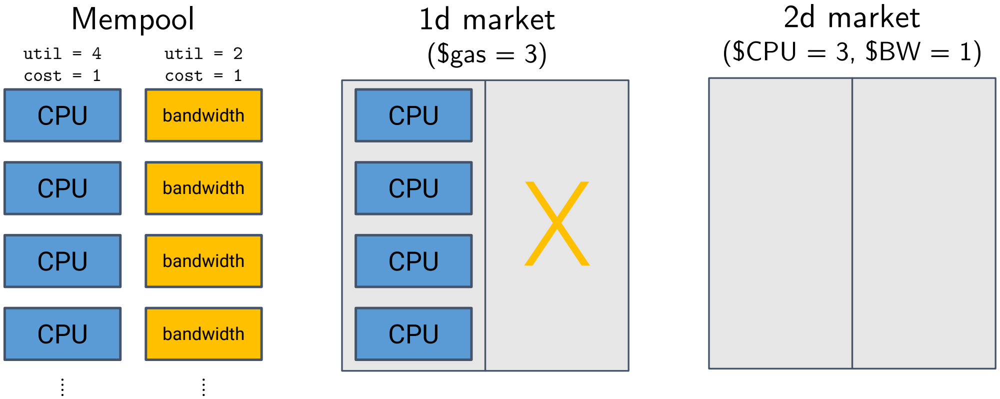

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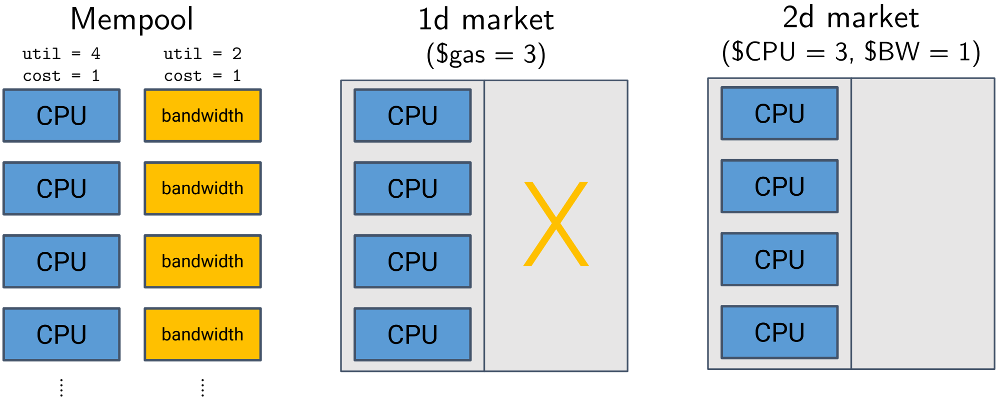

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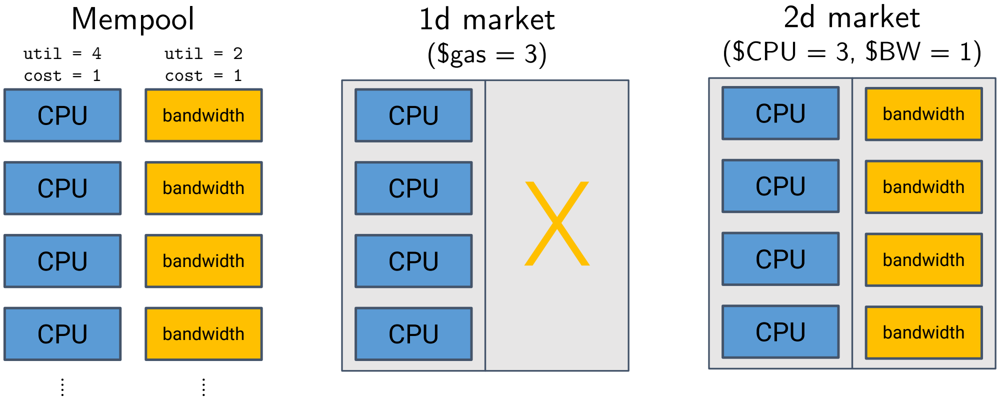

Orthogonal resources should be priced separately

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Let's formalize this

- A **transaction** j consumes a vector of resources $a_j \in \mathbb{R}_+^m$
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- The vector $x \in \{0,1\}^n$ records which of n possible txns are included in a block
  - Entry $x_j = 1$ if tx j is included and 0 otherwise
- The quantity of resources consumed by this block is then

$$y = \sum_{j=1}^{n} x_j a_j = Ax$$

We constrain & charge for each resource used

- Define a **resource consumption target** $b^\star$
  - Deviation from the target is $Ax - b^\star$
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- Define a **resource consumption limit** b
  - Txns included must satisfy $Ax \leq b$
- Charge for usage of each resource (*e.g.*, EIP-1559)
  - Prices p, mean that transaction j costs (this is burned, *i.e.*, this is the base fee)

$$p^T a_j = \sum_{i=1}^{m} p_i (a_j)_i$$

But how do we determine prices?

- We want a few properties:
  - $(Ax)_i = b_i^\star \rightarrow$ no update
  - $(Ax)_i > b_i^\star \rightarrow p_i$ increases
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Is this a good update rule?

Update rules are implicitly solving an optimization problem

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Specific choice of objective by network designer $\implies$ specific update rule

Outline

Setting (for now):

Network designer is omniscient and determines txns in each block

Loss function is network's unhappiness with resource usage

- Network designer determines **loss function** for resource allocation problem; *e.g.*:

$$\ell(y) = \begin{cases} 0 & y = b^\star \\ \infty & \text{otherwise} \end{cases}$$

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We encode all tx constraints in set S

- $S \subseteq \{0, 1\}^n$ is the set of allowable transactions
  - Network constraints, *e.g.*, $Ax \leq b$
  - Interactions among txns, *e.g.*, bidders for MEV opportunity

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- But we will see that the network does not need to know q!

The resource allocation problem

$$
\begin{array}{ll}
\text{maximize} & q^T x - \ell(y) \\
\text{subject to} & y = Ax \\
& x \in S.
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- **Objective:** Maximize utility of included txns minus the loss incurred by the network
- **Constraints:** Utilization y is resource usage of included txns, and x is in the set of allowable txns $S \subseteq \{0,1\}^n$ (can be very complex/hard to solve!)

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  - Doesn't decide which txns are in a block (block builders do this)
  - Doesn't know utilities q

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- Goal: set prices so that this problem is solved optimally on average

Outline

Duality theory: relaxing constraints to penalties

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- We will 'decouple' utilization of network and that of tx producers
- Correctly set penalty $\rightarrow$ dual problem = original problem & utilizations are equal

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- Dual problem is to find the prices p that minimize dual function $g(p)$
- From before, p are the prices for violating prev. constraint $y = Ax$
  - Relaxing constraint to penalty $\rightarrow$ pay per unit violation
- Problem is separable, so $g(p)$ decomposes into two easily interpretable terms:

$$g(p) = \underbrace{\sup_{y} \left(p^T y - \ell(y) \right)}_{\text{network}} + \underbrace{\sup_{x \in S} \left(q - A^T p \right)^T x}_{\text{tx producers}}$$

- Evaluating the 1st term is easy (conjugate function). Let's look at the 2nd...

Second term: block building problem

- Maximize net utility (utility minus cost) subject to tx constraints

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Second term: block building problem

- Maximize net utility (utility minus cost) subject to tx constraints

$$\begin{array}{ll} \text{maximize} & (q - A^T p)^T x \\ \text{subject to} & x \in S. \end{array}$$

- Exact problem solved by block producers! $\to$ Network can observe $x^\star$

What do we get at optimality?

- Let $p^\star$ be a minimizer of $g(p)$, *i.e.*, prices are set optimally
- Assume the block building problem has optimal solution $x^\star$
- The optimality conditions are that 'supply' matches 'demand'

$$\nabla g(p^\star) = y^\star - Ax^\star = 0$$

where $y^\star$ satisfies $\nabla \ell(y^\star) = p^\star$

Key results

1. Prices that minimize g charge the tx producers exactly the marginal costs faced by the network:

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2. These prices incentivize tx producers to include txns that maximize welfare generated $q^T x$ minus the network loss $\ell(Ax)$

Cool. So how do we minimize $g(p)$?

- We can compute the gradient:

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- Network observes $x^\star(p)$ from previous block (block building problem soln)
- Then network applies favorite optimization method (*e.g.*, gradient descent)

$$p^{t+1} = p^t - \eta \nabla g(p^t)$$

Outline

Why are transactions so expensive?

Transactions and resources

The resource allocation problem

Setting prices via duality

Does gradient descent Just Work™?

Let's play a game

- Two players: network and block producers. At block t:

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  2. Users submit txns (with utilities q^t, resources A^t), possibly adversarially!
  3. Network receives payoff $g_t(p^t)$ (from duality)
- Metric: *regret* of the network ('welfare gap')

$$\frac{1}{T}\left(\sum_{t=1}^{T} g_t(p^t) - \min_{p^\star} \sum_{t=1}^{T} g_t(p^\star)\right)$$

- Interpretation: difference between dynamic update rule and the best fixed prices $p^\star$
  - Knowing $p^\star$ requires omniscience: assumes you know all future txns!

Main result:

- Gradient descent price update with fixed step size $\eta = M/B\sqrt{T}$ gives

$$\frac{1}{T}\left(\sum_{t=1}^{T} g_t(p^t) - \min_{p^\star} \sum_{t=1}^{T} g_t(p^\star)\right) \leq \frac{4MB}{\sqrt{T}}$$

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- Online convex optimization shines in this setting (common in blockchains!)
  - Note: does not require that we ever converge to the optimal fixed price $p^\star$

Main result II:

- This scheme is optimal in a certain sense: zero regret on average (with correct step size)
  - Directly from basic online convex optimization results
  - There exists a (stochastic) adversary that matches this bound
  - If utilization is stochastic, prices converge to clearing price

Main result II:

- This scheme is optimal in a certain sense: zero regret on average (with correct step size)
  - Directly from basic online convex optimization results
  - There exists a (stochastic) adversary that matches this bound
  - If utilization is stochastic, prices converge to clearing price
- This result is stronger than 'traditional' game theoretic results:
  - Does not require the adversary to be rational
  - Only requires adversary to be bounded (*e.g.*, have a budget or max block size)
  - Does not require playing to an equilibrium

Some simple examples:

Update rule

$$p^{t+1} = p^t - \eta(b^\star - Ax^\star)$$

Loss function

$$\ell(y) = \begin{cases} 0 & y = b^\star \\ \infty & \text{otherwise} \end{cases}$$

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Update rule	Loss function
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$p^{t+1} = \left(p^t - \eta(b^\star - Ax^\star)\right)_+$	$\ell(y) = \begin{cases} 0 & y \leq b^\star \\ \infty & \text{otherwise} \end{cases}$

Conclusion: choose your objective, not the update rule!

Choice of **objective function** by network designer yields an "optimal" price update rule via our optimization-based framework

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Choice of **objective function** by network designer yields an "optimal" price update rule via our optimization-based framework

No difference between 'correctly' fixing prices with oracle knowledge of future and using online gradient descent algorithm.

These results hold without assumptions of demand distributions or of market-clearing prices!

Extensions and future work

- What should the resources be?
  - How do you optimally trade-off between complexity & ease of use?
  - How do you design a loss function for desired performance characteristics?
  - Implementations by Avalanche and Penumbra teams may provide insights
  - Related to blob pricing and L1 vs L2 gas on rollups

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  - Related to blob pricing and L1 vs L2 gas on rollups
- What update rules are most useful? [Convergence behavior vs. complexity]
- Likely relevant for many similar mechanisms...

For more info, check out our paper!

Paper

Thank you!

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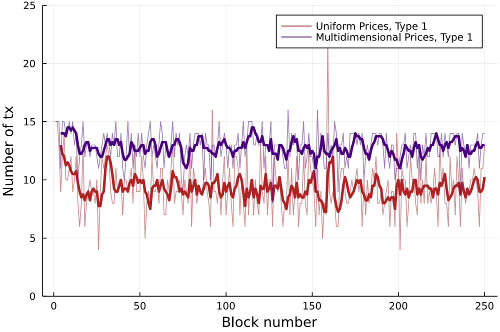
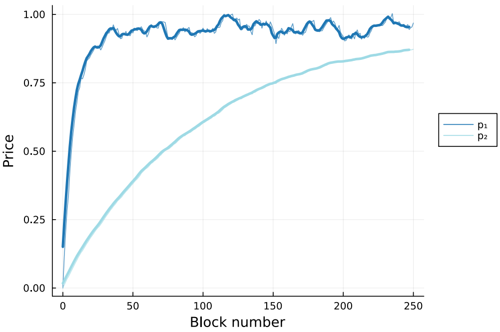
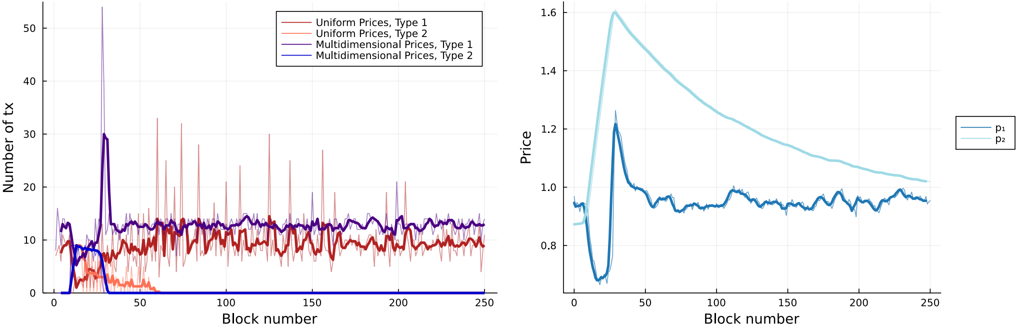
Appendix

Multidimensional fees increase throughput





Even when the tx distribution shifts



And resource utilitaztion better tracks targets